

**Course offered for the PhD program
in Civil, Chemical and Environmental Engineering
a.y. 2024/2025 (XL cycle)**

(course is open for participation of students from other PhD cycles or programs)

1. Title

Large deflection of slender beams: Euler's elastica

2. Course Objectives and Description

The problem of determining the equilibrium states of a thin rod, straight and prismatic in its undistorted reference configuration, which is bent in a principal plane so that the centerline becomes a plane curve (referred to as *elastica*), was first solved by Leonhard Euler (1707-1783) with the advice of Daniel Bernoulli (1700-1782).

The model by Euler is simple and well established, but it is still worthy of dedicated lectures. In fact, this model is widely used in mechanical engineering and soft robotics, as well as for biomedical tools. Additional (unusual) applications can be found in a hot-cutting blade for the fabrication of formworks for concrete shell structures, in devices for timber transportation along rivers and in cattle-catching tools like the lasso, in the optimization of the limbs of the bow, in the illustration of the role played by configurational, or Eshelby-like, forces when the elastic rod constrained within a curved frictionless sleeve.

The course aims to show students the fundamental results on largely inflected elastic beams, also presenting a nonlocal model, based on the recently proposed concept of flexural tensegrity, which falls within the framework of Euler's elastica.

Outline:

- Definition of the problem: inextensible thin rod under large deflection
- Expression of the curvature
- Derivation of field equations from total energy minimization
- The kinetic analogue
- Examples of resolution via elliptic integrals, shape function, numerical integration
- A model of nonlocal elastica based on flexural tensegrity

3. Course Organization

Frontal lecture on both blackboard and electronic devices

4. Teacher

Claudio Boni

5. Duration and credits

6 hours – 1 ECTS

6. Activation mode and teaching period

March 2025 (tentative)

7. Deadline for registration

2 weeks before the first lecture – email message to the teacher

8. Final exam

Home assignment.